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Existence and Uniqueness of Solutions for Extended Fractional Differential Equations

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***Abstract*: In this paper, we have discussed some results on existence and uniqueness theorems for definition of extended fractional differential equation of order α, $0 < \alpha < 1$ and proved theorem existence and uniqueness theorems for definition of Extended fractional differential equations of order $n\alpha, n < \alpha \leq n + 1$ *and* $0 < n\alpha < 1$ and give examples to support results.**

***Keywords*: conformable fractional derivative, extended fractional derivative, existence and uniqueness theorems, Extended sequential Extended fractional differential equations.**

1. Introduction

The majority of the fractional derivatives are defined by means of the fractional integral. All these fractional derivatives do not satisfy the properties of the classical integer order derivatives. We are aware that the integer order derivative of a constant is zero and we expect the same in case of fractional derivatives, but this is not the case in most of the fractional derivatives except the Caputo fractional derivative. Also, the properties of the classical derivatives like the Product rule, the Quotient rule, the Chain rule, Rolle's and Lagrange's Mean value theorems are not satisfies by fractional derivatives (Podlubny, I (1999) [15]., Miller, K. S. and Ross, B (1993) [14]). Few new fractional derivatives, called them the conformable fractional derivatives and proved some of the properties that are not satisfied by the earlier fractional derivatives. All these conformable derivatives are some extensions of the classical limit form definition (Katugampola U. N. 2014 [11], Khalil R. 2014 [12], Abdljawad et al. 2015 [1], Hammad M. et al. 2014[8], Almeida, R. et al 2017.[4])

Kamble at.al. gives the definition of Extended fractional derivative [19] as follows

Extended Fractional Derivative: [19] Definition 3.1: (first form) Let f [0, ∞) -⟶ R be any real valued function and $\alpha \in (0,1], a > 0, a \in R, h > 0$. We define (α, a) Extended fractional Derivative of f of order α at $t \in [0, \infty)$,denoted $D_a{}^{\alpha}f(t) = (f_a)^{\alpha}(t) = T^{\alpha}{}_a f(t)$, by

$$D_a{}^{\alpha}f(t) = (f_a)^{\alpha}(t) = T^{\alpha}{}_a f(t) = \lim_{h\to 0}\frac{f\left(ta^{ht^{-\alpha}}\right) - f(t)}{h}$$

for all $t > 0$, Provided the limit exist.

The maclaurin series expansion of $a^{ht^{-\alpha}}$ is

$$a^{ht^{-\alpha}} = 1 + ht^{-\alpha}lna + O(h)^2$$

(Second form):

$$D_a{}^{\alpha}f(t) = (f_a)^{\alpha}(t) = T^{\alpha}{}_a f(t) = \lim_{h\to 0}\frac{f(t(1 + ht^{-\alpha}lna + O(h)^2)) - f(t)}{h}$$

for all $t > 0$, Provided the limit exist.

Remark 3.1: In case the limit exists, we say that f is (α, a)-extended fractional differentiable (EFD).

If f is (α, a)- extended fractional differentiable in some (0, c), c > 0, and $\lim_{t\to 0^+} f_a{}^{\alpha}(t)$ exists then we define $\lim_{t\to 0^+} f_a{}^{\alpha}(t) = f_a{}^{\alpha}(0)$.

Remark 3.2: It can be observed that the definition 2.1 turns out to be the classical definition of the derivative of first order for the special case α = 1 and a = e.

Remark 3.3: Setting a = e in the definition 2.1, we get the definition of the New Fractional Derivative as defined by U. N. Katugampola in 2014.

Remark: 3.4: The Extended Fractional Derivative is depends on both the order $'\alpha'$ and the point 'a'.

Remark: 3.5: The same order derivative is different for different values of 'a'.

Remark 2.1. If let γ be α, extended, $a > 0, a \in$ R Extended fractional diffferentiable function , $\alpha \in (0,1]$ *then*

$$D_a{}^{\alpha}f(t) = (\ln a)t^{1-\alpha}\frac{df(t)}{dt}$$

This definition follows classical results like the product rule, the quotient rule. The chain rule and the results which are similar to Rolle's theorem and the mean value theorems and obtained the extended fractional derivative of some of the elementary functions like the exponential function, polynomial function and 2 trigonometric functions. and define extended fractional derivative of higher order as follows

(α, a) Extended Fractional Derivative: [19]

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Let f [0, ∞) -⟶ R be any real valued function $\alpha \in (\mathrm{n}, \mathrm{n}+1], \mathrm{n} \in \mathrm{N}, \mathrm{a} > 0, \mathrm{a} \in \mathrm{R}, \mathrm{t} \in [0, \infty)$ $h > 0$. We define (α, a) Extended fractional Derivative of f of order α at $t \in [0, \infty)$,denoted $D_a{}^{\alpha}\mathrm{f(t)} = (\mathrm{f}_a)^{\alpha}(\mathrm{t}) = T^{\alpha}{}_{\mathrm{a}}\mathrm{f(t)}$, by

for $\mathrm{n} < \alpha \leq \mathrm{n}+1$

$$D_a{}^{\alpha}(\mathrm{t}) = \lim_{\mathrm{h}\to 0} \frac{D^{\lceil\alpha\rceil-1}\left(\mathrm{t} + \mathrm{h}(\ln a)\mathrm{t}^{(\lceil\alpha\rceil-\alpha)}\right) - D^{\lceil\alpha\rceil-1}(\mathrm{t})}{\mathrm{h}}$$

Where ⌈α⌉ is the smallest integer greater than or equal to α.

$$(\lceil\alpha\rceil = n+1)$$

or

$$\mathrm{D}_a{}^{\alpha}(\mathrm{f})(\mathrm{t}) = \lim_{\mathrm{h}\to 0} \frac{D^{\mathrm{n}}\left(\mathrm{t} + \mathrm{h}\,(\ln a)\,\mathrm{t}^{(\mathrm{n}+1-\alpha)}\right) - D^{\mathrm{n}}(\mathrm{t})}{\mathrm{h}},$$

Where D^{n} is the n-th order ordinary derivative at t if γ is α-comformable differentiable in some (0, b), b >0 and $\lim_{\tau\to 0^+} \gamma^{\alpha}(\tau)$,exists then define $\lim_{\tau\to 0^+} \gamma^{\alpha}(\tau) = \gamma^{\alpha}(0)$.

Remark: 3.11 [19] follows directly from definition

$$\mathrm{T}^{\alpha}{}_{\mathrm{a}}(\mathrm{f}) = (\ln a)\mathrm{t}^{(\lceil\alpha\rceil-\alpha)} \frac{\mathrm{d}^{\lceil\alpha\rceil}\mathrm{f}}{(\mathrm{dt})^{\lceil\alpha\rceil}}$$

where we assume that n < α ≤ n+1, n ∈ N and γ is (n+1) times α- exponential differentiable at τ > 0, $\lceil\alpha\rceil = \mathrm{n}+1$ is the smallest integer greater than or equal to α.

Theorem 3.3: [19] (α, a) Extended Fractional Derivative of Some Elementary Functions.

Let $\alpha \in (0,1), \mathrm{a} > 0, \mathrm{a} \in \mathrm{R}, \lambda \in \mathrm{R}$ then

1) $D_a{}^{\alpha}(\mathrm{t}^{\lambda}) = \lambda \mathrm{t}^{\lambda-\alpha}(\ln a)\ \forall\, \lambda \in \mathrm{R}$
2) $D_a{}^{\alpha}(\lambda) = 0$
3) $D_a{}^{\alpha}(\lambda) = 0, \forall\, \lambda \in \mathrm{R}$
4) $D_a{}^{\alpha}(\mathrm{e}^{\lambda \mathrm{t}}) = \lambda \mathrm{t}^{1-\alpha}(\ln a)\mathrm{e}^{\lambda \mathrm{t}}, \lambda \in \mathrm{R}$
5) $D_a{}^{\alpha}(\mathrm{a}^{\lambda \mathrm{t}}) = \lambda \mathrm{t}^{1-\alpha}(\ln a)^2 \mathrm{a}^{\lambda \mathrm{t}}, \lambda \in \mathrm{R}$
6) $D_a{}^{\alpha}(\sin\lambda \mathrm{t}) = \lambda \mathrm{t}^{1-\alpha}(\ln a)\cos\lambda \mathrm{t}, \lambda \in \mathrm{R}$
7) $D_a{}^{\alpha}(\cos\lambda \mathrm{t}) = -\lambda \mathrm{t}^{1-\alpha}(\ln a)\sin\lambda \mathrm{t}, \lambda \in \mathrm{R}$
8) $D_a{}^{\alpha}(\tan(\lambda \mathrm{t})) = \lambda \mathrm{t}^{1-\alpha}(\ln a) sec^2(\lambda \mathrm{t}), \lambda \in \mathrm{R}$
9) $D_a{}^{\alpha}(\cot(\lambda \mathrm{t})) = -\lambda \mathrm{t}^{1-\alpha}(\ln a)\mathrm{cosec}^2(\lambda \mathrm{t}), \lambda \in \mathrm{R}$
10) $D_a{}^{\alpha}(\sec(\lambda \mathrm{t})) = \lambda \mathrm{t}^{1-\alpha}(\ln a)\sec(\lambda \mathrm{t})\tan(\lambda \mathrm{t})\,, \lambda \in \mathrm{R}$
11) $D_a{}^{\alpha}(\mathrm{Cosec}(\lambda \mathrm{t})) = -\lambda \mathrm{t}^{1-\alpha}(\ln a)\mathrm{Cosec}(\lambda \mathrm{t})\cot(\lambda \mathrm{t}), \lambda \in \mathrm{R}$

We can continue our definition of extended fractional integration in a natural way based on the classical integration

Definition 4.1: [19] (α, a) Extended fractional integral:

let α ∈ (0, 1] and $t > 0, a > 0, a \in R$ let f be a function defined on (0, ∞] then the (α, a) Extended fractional integral is defined by,

$$I_a{}^{\alpha}\mathrm{f(t)} = \int_0^{\mathrm{t}} \frac{\mathrm{f(s)}}{(\ln a)\,\mathrm{s}^{1-\alpha}} \mathrm{ds}$$

where t ≥ 0 provide the integral exist.

Now we have enough material to prove our main results in the following sections.

2. Results and Discussion

Existence and uniqueness theorem on Extended definition fractional differential equations:

Kamble at al give definition of Extended fractional derivative [19].

Basic theory of differential equations is given in 'A second course in Elementary Ordinary Differential Equations' (Finan M 2013). [7]

A new definition of fractional derivative is introduced known as conformable fractional derivative (Khalil R., Horani M. Al, Yousef A., and Sababheh M., 2014) [12]

Remarks On conformable fractional calculus is given (Abdeljawad T.,2015) [1]

A new fractional derivative with classical properties is proved similar to conformable fractional derivative (Katugampola U. N., 2014) [11]

Considering the different initial and boundary conditions and by the applications of a variety of fixed point theorems, many authors have proved the existence and uniqueness results.

In [16] Kamble Rajratna M, Kulkarni Pramod Ramakant, have derived "Laplace Transform and Laplace Decomposition Method For Of order (α,1) Fractional Differential Difference Equations With Interval Conditions Linear And Nonlinear".

In [17] Kamble Rajratna M.,Kulkarni Pramod Ramakant ,have proved "Existence and Uniqueness of Continuous Solutions for Conformable Fractional Integro-Differential" Equations in Cone Metric Spaces".

In [18] Kamble, Rajratana, and Pramod Kulkarni. "Numerical Solutions of the SIR Mathematical Model of Computer Viruses Involving Non-linear Fractional Order Differential Equation".

In [19] Rajratana, Maroti Kamble, and P.R. Kulkarni Given new definition of fractional derivative "Extended fractional derivative: Some results involving classical properties and applications".

In [20] Kamble Rajratna M., and Pramod R. Kulkarni have proved "Existence and uniqueness of solutions for exponential fractional differential equations".

In [21] Kamble Rajratna, M., et al. "Gheorghe Săvoiu, Mladen Čudanov and Vesna Tornjanski." have proved "On Some Existence and Uniqueness Results For Nonlinear Fractional Differential Equations With Boundary Conditions"

In all the paper any Extended fractional derivative lies in (0,1)

Theorem 3.1:

Suppose

$$\frac{t^{\alpha-1}}{lna}p(\tau), \frac{t^{\alpha-1}}{lna}Q(\tau)\ are\ continous\ functions\ on\ I = [a,b] and\ f\ be\ \alpha - Extended\ fractional\ differentiable\ with\ \alpha \in (0,1], t_0 \in I\ then\ the\ solution\ of\ initial\ value\ problem (I.V.P)$$

$$D^{\alpha}f + P(t)f = Q(t) \ldots (5)$$

$f(t_0) = f_0$…(6)

Has unique solution on I, $t_0 \in I$

Proof: The extended fractional derivative of γ at τ is given as follows

$D_a{}^{\alpha}\mathrm{f(t)} = (\mathrm{f}_a)^{\alpha}(\mathrm{t}) = T^{\alpha}{}_{\mathrm{a}}\mathrm{f(t)} = \lim_{h\to 0}\frac{\mathrm{f}(\mathrm{t}(1+ht^{-\alpha}lna+O(h)^2))-\mathrm{f(t)}}{\mathrm{h}}$ …(7)

provided the limit exists. It can be verified that

$$D_a{}^{\alpha}\mathrm{f(t)} = (\ln a)\mathrm{t}^{1-\alpha}\frac{\mathrm{df(t)}}{\mathrm{dt}}$$

Hence the differential equation (5)can be expressed as $\frac{\mathrm{df(t)}}{\mathrm{dt}} + p(t)(f)(t) = q(t)$

$\frac{df(t)}{dt} + \frac{t^{\alpha-1}}{lna}P(t)(f)(t) = \frac{t^{\alpha-1}}{lna}q(t)$…(9)

Since $\frac{t^{\alpha-1}}{lna}p(\tau), \frac{t^{\alpha-1}}{lna}Q(\tau)$ are continuous functions defined on the interval I = [a,b],a,b ∈ R,a > 1 and $\frac{\mathrm{t}^{(1-\alpha)}}{lna}$ is continuous, it follows from the existence and uniqueness theorem for ordinary differential equations that the initial value problem given by equations (9) and (6) has a unique solution. This proves that the initial value problem (5) (6) has a unique solution. The unique and the unique solution is given by

$$\frac{df(t)}{dt} = \frac{\mathrm{t}^{(\alpha-1)}}{lna}[Q(t) - P(t)(f)(t)]$$
$$f(t) = f(t_0) + \int_{t_0}^{t}\frac{\mathrm{t}^{(\alpha-1)}}{lna}[Q(t) - P(t)(f)(t)]dt$$

Theorem 3.2:

$\frac{t^{\alpha-1}}{lna}P_{(n-1)}(t), \frac{t^{\alpha-1}}{lna}P_{(n-2)}(t), \ldots, \frac{t^{\alpha-1}}{lna}P_{(1)}(t), \frac{t^{\alpha-1}}{lna}P_{(0)}(t) are\ continous\ functions\ on\ I = [a,b]$

$n\alpha \in (0,1]$.

$then\ the\ solution\ of following\ \ initial\ value\ problem$

$D^{n\alpha}f + P_{n-1}(t)D^{(n-1)\alpha})f \ldots + P_2(t)(D^{2\alpha})f + P_1(t)D^{\alpha}f + P_0(t)f = Q(t)$ ...(10)

$f(t_0) = f_0,\ D^{\alpha}(f)(t_0) = f_{1,\ldots,}D^{(n-1)\alpha})f(t_0) = f_{n-1}\quad {}_{t_0\in I=(a,b),a\geq 0}$…(11)

Possesses unique solution

Proof: let $\mu_1 = f(t), \mu_2 = D^{\alpha}f(t),\ \mu_3 = D^{2\alpha}f(t), \mu_n = D^{(n-1)\alpha}f(t)$, here $\mu_1\ is\ the\ solution.$

Therefore, we have

$$D^{\alpha}\mu_1 = \mu_2$$

$D^{\alpha}\mu_2 = \mu_3$…

$$D^{\alpha}\mu_{n-1} = \mu_n$$

Also, from the equation (10)

$$D^{\alpha}\mu_n = -P_{n-1}(t)\mu_n - \cdots - P_2(t)\mu_3 - P_1(t)\mu_2 - P_0(t)\mu_1 + Q(t)$$

The Above system can be written as

$$D^{\alpha}\begin{bmatrix}\mu_1\\ \mu_2\\ .\\ .\\ .\\ \mu_{n-1}\\ \mu_n\end{bmatrix} + \begin{bmatrix}0, & -1, & 0 & 0\ldots 0\\ 0 & ,0,-1 & ,0,\ldots 0 & \\ & & . & \\ & & . & \\ & & . & \\ 0 & ,0, & 0, & 0,\ldots\ldots -1\\ p_{0,} & p_{1,} & p_{2,} & p_{3,}\ldots.p_{n-1,}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ .\\ .\\ .\\ \mu_{n-1}\\ \mu_n\end{bmatrix} = \begin{bmatrix}0\\ 0\\ .\\ .\\ .\\ 0\\ q(t)\end{bmatrix}$$

$$D^{\alpha}U(t) + R(t)U(t) = S(t)$$

where

$$U(t) = \begin{bmatrix}\mu_1\\ \mu_2\\ .\\ .\\ .\\ \mu_{n-1}\\ \mu_n\end{bmatrix}, R(t) = \begin{bmatrix}0, & -1, & 0 & 0\ldots 0\\ 0 & ,0,-1 & ,0,\ldots 0 & \\ & & . & \\ & & . & \\ & & . & \\ 0 & ,0, & 0, & 0,\ldots -1\\ p_{0,} & p_{1,} & p_{2,} & p_{3,}\ldots p_{n-1,}\end{bmatrix},$$

$$\mathrm{S}(t) = \begin{bmatrix}0\\ 0\\ .\\ .\\ .\\ 0\\ q(\tau)\end{bmatrix},$$

$$\Gamma(t_0) = \begin{bmatrix}f(t_0) = \mu_1(t_0)\\ D^{\alpha}(f)(t_0) = \mu_2(t_0)\\ .\\ .\\ .\\ D^{(n-2)\alpha})(f)(t_0) = \mu_{n-1}(t_0)\\ D^{(n-1)\alpha})(f)(t_0) = \mu_n(t_0)\end{bmatrix} = \Gamma(t)\begin{bmatrix}\mu_1(t_0) = f_0\\ \mu_2(t_0) = f_1\\ .\\ .\\ .\\ \mu_{n-1}(t_0) = f_{n-2}\\ \mu_n(t_0) = f_{n-1}\end{bmatrix}$$

$U(t)' + \frac{\mathrm{t}^{(\alpha-1)}}{lna}[R(t)U(t)] = \frac{\mathrm{t}^{\alpha-1}}{lna}S(t)$…(12)

And the initial condition changed to $\Gamma(t_0) = \Gamma(\mathrm{t})$...(13)

By theorem in Finan M., A second course in Elementary Ordinary Differential Equations, it follows that the initial value problem (12) (13) has a unique solution. Hence the initial value problem (10) (11) has a unique solution.

Example 1: As an application of the results proved so far, we will solve an initial value problem. $D^{0.5}f(t)+t^{-\frac{1}{2}}(lna)^2 f(t)=ta^{-t}$ with the initial conditions

$$\gamma(0)=0$$

Proof: First we note that

$$D_a{}^{\alpha}\mathrm{f(t)}=(\ln a)\mathrm{t}^{1-\alpha}\frac{\mathrm{df(t)}}{\mathrm{dt}}$$

$$D_a{}^{\alpha}\left(\mathrm{a}^{\lambda\mathrm{t}}\right)=\lambda\mathrm{t}^{\lambda-\alpha}(\ln a)^2\mathrm{a}^{\lambda\mathrm{t}},\lambda\in\mathrm{R}$$

Since the Extended fractional derivative satisfies the product rule. We can apply to differential equation to solve differential equations.

$$D^{0.5}(\gamma(\tau)a^{\tau})=\tau$$

$$(\gamma(\tau)a^{\tau})=\int_0^{\tau}\frac{1}{(lna)}t^{-\frac{1}{2}}d\tau$$

$\left(\gamma(\tau)\right)=\frac{1}{(lna)}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}a^{-t}+ca^{-t}$ (since by using definition of Extended fractional integral)

After applying initial condition we get value of c and we get the unique Solution.

$$\left(\gamma(\tau)\right)=\frac{1}{(lna)}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}a^{-t}$$

3. Conclusion

In this paper, we have discussed some results on existence and uniqueness theorems for definition of Extended fractional differential equation of order $\alpha,\ 0<\alpha<1$ and proved theorem existence and uniqueness theorems for definition of Extended fractional differential equations of order $\mathrm{n}\alpha, n<\alpha\leq n+1\ and\ 0<n\alpha<1$ and give examples to support results.

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