New Generalized Definition of Conformable Fractional Derivative

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***Abstract*: In this paper we find new generalized definition of conformable fractional derivative. The definition satisfies linearity property, product rule for derivative, quotient rule for derivative and derivative of constant is zero. compare the chain rule formula of ordinary derivative and chain rule satisfied by new generalized definition of conformable fractional derivative. chain rule for like first order derivative are not satisfied. This can be proved by giving example and find derivatives of some standard functions by new generalized definition of fractional derivative. We introduced new generalized definition of conformable fractional derivative. We conclude that this definition coinside with the classical definition of derivative $\alpha = 1$. We introduce definition For $\alpha \in (0, 1]$ and generalize for any $\alpha \in (n, n+1]$.**

***Keywords*: chain rule, chain rule for new generalized conformable fractional derivative, fractional derivative, new generalized definition of conformable fractional derivative.**

1. Introduction

The derivative is known to all who knows elementary calculus.

n^{th} *derivative of f is* $\frac{D^n f(x)}{dx^n}$ where n is positive integer.

When L Hopital asked the question to Leibniz what is the meaning when n is fraction.

And begins fractional calculus.

Two of which are the most popular definition of fractional derivative is.

1) Riemann liouville definition. For $\alpha \in [n-1, n)$ the α derivative of f is
$$D_a{}^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}} dx$$

2) Coputo definition. For $\alpha \in [n-1, n)$ the α derivative of f is
$$D_a{}^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(x)}{(t-x)^{\alpha-n+1}} dx$$

Definitions including (i) and (ii) above satisfy the property that the fractional derivative is linear.

This is the only property satisfied from the first derivative by all of the definitions.

we can define our new generalized definition of conformable fractional derivative for $\alpha \in (0,1]$ and generalize this definition for any $\alpha \in (n, n+1]$.

However, the other properties of fractional derivative such as product rule of derivative and derivative of constant is zero and quotient or division rule for derivative, derivative of chain rule is not satisfied by most of other definitions:

i. The Riemann–Liouville derivative definition *does not* satisfy $D_a{}^{\alpha}(1) = 0$ but Caputo derivative satisfies $D_a{}^{\alpha}(1) = 0$, if α is not a natural number.
ii. All fractional derivatives *do not* satisfy the formula of the derivative of the product of two functions:
iii. All fractional derivatives *do not* satisfy of the formula derivative of the quotient or division of two functions:

- All fractional derivatives *do not* satisfy the chain rule:
$$D_a{}^{\alpha}(f \circ g) = f^{\alpha}(g(t)) g^{\alpha}(t)$$
- All fractional derivatives *do not satisfy*:
$$D^{\alpha} D^{\beta} f = D^{\alpha+\beta} f \text{ general.}$$

2. Methods and Approach

Definition 1: $f: [0, \infty) \to R$ *and* $t > 0$ then the first order derivative from first principle is given by,

$\frac{df}{dt} = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon) - f(t)}{\varepsilon}$ in this definition a forward difference $f(t+\varepsilon) - f(t)$ is used.

This definition Satisfies the following properties,

i. $\frac{d}{dt}(af + bg) = a\frac{d}{dt}(g) + b\frac{d}{dt}(f)$, for all $a, b \in$ R and f, g in the domain of T_1.
ii. $\frac{d}{dt}(t^p) = pt^{p-1}$
iii. $\frac{d}{dt}(fg) = f\frac{d}{dt}(g) + g\frac{d}{dt}(f)$
iv. $\frac{d}{dt}\left(\frac{f}{g}\right) = \frac{g\frac{d}{dt}(f) - f\frac{d}{dt}(g)}{g^2}$
v. $\frac{d}{dt}(\lambda) = 0$, for all constant functions $f(t) = \lambda$.

Now in a similar way we can define our new generalized definition of comformable fractional derivative for $\alpha \in (0,1]$ and generalize this definition for any $\alpha \in (n, n+1]$.

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3. Results and Discussion

New generalized definition of conformable fractional derivative:

Now we can define our new generalized definition of conformable fractional derivative for $\alpha \in (0,1]$ and generalize this definition for any $\alpha \in (n, n+1]$.

Definition 2. Given a function $f : [0, \infty) \longrightarrow$ R. Then the *fractional derivative*'' of f of order $\alpha \in (0,1]$ is defined by,

$$(f)^{\alpha}(t) = T_{\alpha} f(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

for all $t > 0$, $\alpha \in (0, 1)$. If f is α-differentiable in some $(0, a)$, $a > 0$, and $\lim_{t \to 0^+} f^{\alpha}(t)$ exists then define $\lim_{t \to 0^+} f^{\alpha}(t) = f^{\alpha}(0)$

Now the two limits

$$\lim_{\varepsilon \to 0^+} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$\lim_{\varepsilon \to 0^-} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Are same as $\varepsilon \to 0^+$ *and* $\varepsilon \to 0^-$, $e^{-\left|\frac{1}{\varepsilon}\right|} \to 0$, but domain of is : $[0, \infty)$ *so the condition* $\varepsilon \to 0^-$ *does not occure.*

We can derive definition for first order from this definition by putting $\alpha = 1$, so $\alpha = 1$ the definition 1 becomes,

$$(f)'(t) = T_1 f(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-1}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$(f)'(t) = T_1 f(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

putting $h = e^{-\left|\frac{1}{\varepsilon}\right|}$ *and as* $\varepsilon \to 0, h \to 0$ so the definition becomes,

$$(f)'(t) = T_1 f(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

Theorem 2.1. If a function $f : [0, \infty) \longrightarrow$ R *is α-differentiable at* $t_0 > 0$, $\alpha \in (0, 1]$, *then f is continuous at* t_0.

Proof:

Since $f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t) = \frac{f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t_0)}{e^{-\left|\frac{1}{\varepsilon}\right|}} \times e^{-\left|\frac{1}{\varepsilon}\right|}$ then

$$\lim_{\varepsilon \to 0} f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t_0)$$

$$= \lim_{\varepsilon \to 0} \frac{f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t_0)}{e^{-\left|\frac{1}{\varepsilon}\right|}} . \lim_{\varepsilon \to 0} e^{-\left|\frac{1}{\varepsilon}\right|}$$

$$\lim_{\varepsilon \to 0} f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t_0) = f^{\alpha}(t_0). 0$$

$$\lim_{\varepsilon \to 0} f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) - f(t_0) = 0$$

Which implies that

$$\lim_{\varepsilon \to 0} f\left(t_0 + e^{-\left|\frac{1}{\varepsilon}\right|} {t_0}^{1-\alpha}\right) = f(t_0)$$

Hence f is continuous.

Theorem.2.2. Let $\alpha \in (0, 1]$ *and f, g be α-differentiable at a point* $t > 0$. *Then*

i. $T_{\alpha}(af + bg) = aT_{\alpha}(f) + bT_{\alpha}(g)$, for all $a, b \in$ R and f, g in the domain of T_{α}.
ii. $T_{\alpha}(af - bg) = aT_{\alpha}(g) - bT_{\alpha}(f)$, for all $a, b \in$ R and f, g in the domain of T_{α}.
iii. $T_{\alpha}(fg) = fT_{\alpha}(g) + gT_{\alpha}(f)$
iv. $T_{\alpha}(f/g) = \frac{gT_{\alpha}(f) - fT_{\alpha}(g)}{g^2}$
v. If in addition , f is differentiable then $T_{\alpha}(f)(t) = t^{1-\alpha} \frac{d}{dt} f(t)$

$$T_{\alpha}(af + bg)$$

$$= \lim_{\varepsilon \to 0} \frac{af\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) + bg\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - \{af(t) + bg(t)\}}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon \to 0} \frac{af\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - af(t) - \{bg\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - bg(t)\}}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon \to 0} \frac{a\left\{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)\right\} + b[g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - g(t)]}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= a \lim_{\varepsilon \to 0} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}} + b . \lim_{\varepsilon \to 0} \frac{g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - g(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= aT_{\alpha} f + b.T_{\alpha} g.$$

Similarly, ii can be proved.

$$T_{\alpha}(fg)(t) = \lim_{\varepsilon \to 0} \frac{f(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}) g(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}) - f(t)g(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon \to 0} \frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)g(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}) + f(t)g(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}) - f(t)g(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon \to 0} \left(\frac{f\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}\right) . g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right)$$

$$+ \lim_{\varepsilon \to 0} \left(\frac{g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) - g(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}\right) . f(t)$$

$$== T_{\alpha}(f) \lim_{\varepsilon \to 0} g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) + f(t)\, T_{\alpha}(g)$$

Since g is continuous $\lim_{\varepsilon \to 0} g\left(t + e^{-\left|\frac{1}{\varepsilon}\right|} t^{1-\alpha}\right) = g(t)$.

Simillarly iv can be proved.

$$T_{\alpha}(f)(t) = t^{1-\alpha} \frac{d}{dt} f(t)$$

$$\lim_{\varepsilon\to 0}\frac{f\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-f(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Let $=h=e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}$ *implies* $e^{-\left|\frac{1}{\varepsilon}\right|}=ht^{\alpha-1}$ *as* $\varepsilon\to 0, h\to 0$

$$=\lim_{h\to 0}\frac{f(t+h)-f(t)}{ht^{\alpha-1}}$$
$$=\lim_{h\to 0}\frac{f(t+h)-f(t)}{ht^{\alpha-1}}$$
$$=t^{1-\alpha}\frac{d}{dt}f(t).$$

Theorem: 2.3 Suppose I And J be open intervals in R, for $t\in I$, let $f: I\to J$ *and* $g: J\to R$ be two functions and f *is* α differentiable at t and g *is* α *at* $f(t)$.then $(gof)(t)$ is α *differentiable* at t and $(gof)^{\alpha}(t)=g'(f(t))f^{\alpha}(t)$.

By definition $(gof)^{\alpha}(t)=\lim_{\varepsilon\to 0}\frac{(gof)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(gof)(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$

Multiplied by $\frac{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}$ then

$$(gof)^{\alpha}(t)=\lim_{\varepsilon\to 0}\frac{g(f\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right))-(g(f)(t))}{e^{-\left|\frac{1}{\varepsilon}\right|}}\times\frac{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}$$

$$(gof)^{\alpha}(t)=\lim_{\varepsilon\to 0}\frac{g(f\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right))-(g(f)(t))}{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}\times\frac{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Since $f^{\alpha}(t)=\lim_{\varepsilon\to 0}\frac{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$ put $\beta=(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)$ and then substitute

$$\varepsilon\to 0 \text{ implies } \beta\to 0$$

into $=\lim_{\varepsilon\to 0}\frac{g(f\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right))-(g(f)(t))}{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}=g'(f(t))$

$$(gof)^{\alpha}(t)=\lim_{\beta\to 0}\frac{g(f(t)+\beta)-(g(f)(t))}{\beta}\times\lim_{\varepsilon\to 0}\frac{(f)\left(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-(f)(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$(gof)^{\alpha}(t)=g'(f(t))f^{\alpha}(t)$$

So the definition does not satisfies the chain rule which is $(gof)^{\alpha}(t)=g^{\alpha}(f(t))f^{\alpha}(t)$.

For example $f(t)=t+1, g(t)=t^2$

$$f^{\frac{1}{2}}(t)=t^{\frac{1}{2}},\qquad g^{\frac{1}{2}}(t^2)=2t^{\frac{3}{2}}$$

$$gof(t)=(t+1)^2,\qquad (gof)^{\frac{1}{2}}(t)=2t^{\frac{1}{2}}(t+1)$$

but

$$g^{\frac{1}{2}}(f(t))f^{\frac{1}{2}}(t)=2(t+1)^{\frac{3}{2}}t^{\frac{1}{2}},$$

Hence

$$2t^{\frac{1}{2}}(t+1)\neq 2(t+1)^{\frac{3}{2}}t^{\frac{1}{2}},$$

$$g^{\frac{1}{2}}(f(t))f^{\frac{1}{2}}(t)\neq (gof)^{\frac{1}{2}}(t)$$

New generalized definition of conformable fractional derivatives of certain functions

1. $T_{\alpha}(t^p)=pt^{p-\alpha}$ for all p$\in R$
2. $T_{\alpha}(1)=0$
3. $T_{\alpha}(e^{cx})=cx^{1-\alpha}e^{cx}, c\in R$
4. $T_{\alpha}(sinbx)=bx^{1-\alpha}cosbx, b\in R$
5. $T_{\alpha}(cosbx)=-bx^{1-\alpha}sinbx, b\in R$
6. $T_{\alpha}(\tan(at))=at^{1-\alpha}sec^2(at), a\in R$
7. $T_{\alpha}(\cot(at))=-at^{1-\alpha}cosec^2(at), a\in R$
8. $T_{\alpha}(\sec(at))=at^{1-\alpha}sec(at)\tan(at), a\in R$
9. $T_{\alpha}(\sec(at))=at^{1-\alpha}sec(at)\tan(at), a\in R$
10. $t_{\alpha}\left(\frac{1}{\alpha}t^{\alpha}\right)=1$

1) Ans:

$$T_{\alpha}(t^p)=\lim_{\varepsilon\to 0}\frac{(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha})^p-t^p}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$=\lim_{\varepsilon\to 0}\frac{t^p+\quad pt^{p-1}e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}+\quad\frac{P(P-1)}{2}t^{p-2}(e^{-\left|\frac{1}{\varepsilon}\right|})^2t^{(1-\alpha)^2}+\dots\dots\dots\dots\dots\dots\dots\dots\dots+-t^p}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$=\lim_{\varepsilon\to 0}\frac{pt^{p-1}e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}+\quad\frac{P(P-1)}{2}t^{p-2}(e^{-\left|\frac{1}{\varepsilon}\right|})^2t^{(1-\alpha)^2}+\cdots\dots\dots\dots}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$=\lim_{\varepsilon\to 0}pt^{p-1}t^{1-\alpha}+\frac{\frac{P(P-1)}{2}t^{p-2}((e^{-\left|\frac{1}{\varepsilon}\right|})^2t^{(1-\alpha)^2}+\cdots\dots\dots\dots}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$=\;pt^{p-\alpha}.$$

2) *Ans.:*

$$T_\alpha(e^{cx}) = \lim_{\varepsilon\to 0} \frac{e^{\left(ct+ce^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right)-e^{ct}}}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$T_\alpha(e^{cx}) = \lim_{\varepsilon\to 0} \frac{e^{ct}e^{ce^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}} - e^{ct}}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Here h=$h = e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha} \Rightarrow e^{-\left|\frac{1}{\varepsilon}\right|} = h.t^{\alpha-1}$

Putting the values

$$T_\alpha(e^{ct}) = \lim_{\varepsilon\to 0} \frac{e^{ct}(e^{ch}-1)}{h.t^{\alpha-1}}$$

$$T_\alpha(e^{cx}) = \lim_{h\to 0} \frac{e^{ct}(e^{ch}-1).c}{c.h.t^{\alpha-1}}$$

$$T_\alpha(e^{cx}) = \frac{e^{ct}c}{t^{\alpha-1}} \lim_{h\to 0} \frac{(e^{ch}-1).}{c.h.}$$

$$as\ \varepsilon \to 0, \Rightarrow h \to 0$$

$$T_\alpha(e^{ct}) = \frac{e^{ct}c}{t^{\alpha-1}}.1$$

$$T_\alpha(e^{ct}) = ct^{1-\alpha}e^{ct}, c \in R$$

3)

$$T_\alpha(sinbt) = bt^{1-\alpha}cosbt, b \in R$$

By using definition.

$$= \lim_{\varepsilon\to 0} \frac{sinb\left(t + e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}\right) - sinbt}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon\to 0} \frac{2cosb\left(t + \frac{e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}}{2}\right)sin\frac{be^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}}{2}}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= cosbx \lim_{\varepsilon\to 0} \frac{sin\frac{be^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}}{2}}{\frac{be^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha}}{2}}.bt^{1-\alpha}$$

$$= cosbt.\, bt^{1-\alpha}$$

Similarly *as above* $T_\alpha(costx) = -bt^{1-\alpha}sinbt, b \in R$ *can be proved.*

$$T_\alpha(\tan(at)) = at^{1-\alpha}sec^2(at), a \in R$$

$$T_\alpha(\tan(at)) = T_\alpha\left(\frac{\sin(at)}{\cos(at)}\right)$$

$$T_\alpha(\tan(at)) = \left(\frac{\cos(at)\,T_\alpha \sin(at) - \sin(at)T_\alpha\cos(at)}{cos^2(at)}\right)$$

$$T_\alpha(\tan(at)) = \left(\frac{\cos(at)\,at^{1-\alpha}\cos(at) - \sin(at)(-at^{1-\alpha}\sin(at))}{cos^2(at)}\right)$$

$$T_\alpha(\tan(at)) = \left(\frac{at^{1-\alpha}cos^2(at) + at^{1-\alpha}sin^2(at)}{cos^2(at)}\right)$$

$$T_\alpha(\tan(at)) = at^{1-\alpha}(1 + tan^2(at))$$

$$T_\alpha(\tan(at)) = at^{1-\alpha}sec^2(at)$$

Similarly $T_\alpha(\cot(at)) = -at^{1-\alpha}cosec^2(at), a \in R$,

4)

$$T_\alpha(\sec(at)) = at^{1-\alpha}sec(at)\tan(at), a \in R$$

$$T_\alpha(\sec(at) = T_\alpha(\frac{1}{\cos(at)})$$

$$T_\alpha(\sec(at) = \frac{-1(T_\alpha(\cos(at))}{cos^2(at)}$$

$$T_\alpha(\sec(at) = \frac{-1(-at^{1-\alpha}\sin(at))}{cos^2(at)}$$

$$T_\alpha(\sec(at) = \frac{(at^{1-\alpha}\sin(at))}{cos(at)}\frac{1}{\cos(at)}$$

$$T_\alpha(\sec(at)) = at^{1-\alpha}sec(at)\tan(at), a \in R$$

Similarly,

$$T_\alpha(cosec) = -at^{1-\alpha}cosec(at)\cot(at), a \in R$$

5)

$$T_\alpha\left(\frac{1}{\alpha}t^\alpha\right) = \lim_{\varepsilon\to 0} \frac{\frac{1}{\alpha}(t + e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha})^\alpha - \frac{1}{\alpha}t^\alpha}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon\to 0} \frac{\frac{1}{\alpha}(t^\alpha + \alpha t^{\alpha-1}e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha} + \frac{\alpha(\alpha-1)}{2}t^{\alpha-2}(e^{-\left|\frac{1}{\varepsilon}\right|})^2t^{(1-\alpha)^2} + \cdots\ldots\ldots\ldots\ldots\ldots\ldots) - \frac{1}{\alpha}t^\alpha}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon\to 0} \frac{\frac{1}{\alpha}(\alpha t^{\alpha-1}e^{-\left|\frac{1}{\varepsilon}\right|}t^{1-\alpha} + \frac{\alpha(\alpha-1)}{2}t^{\alpha-2}(e^{-\left|\frac{1}{\varepsilon}\right|})^2t^{(1-\alpha)^2} + \cdots\ldots\ldots)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon\to 0}\frac{1}{\alpha}(\alpha t^{\alpha-1}t^{1-\alpha}) + \frac{\frac{1}{\alpha}(\frac{\alpha(\alpha-1)}{2}t^{\alpha-2}\left((e^{-\left|\frac{1}{\varepsilon}\right|}\right)^2t^{(1-\alpha)^2} + \cdots\ldots\ldots)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

$$= \lim_{\varepsilon\to 0}\frac{1}{\alpha}(\alpha t^{\alpha-1}t^{1-\alpha}) + \lim_{\varepsilon\to 0}\frac{\frac{1}{\alpha}\left(\frac{\alpha(\alpha-1)}{2}t^{\alpha-2}\left((e^{-\left|\frac{1}{\varepsilon}\right|}\right)^2 t^{(1-\alpha)^2}+\cdots\ldots\ldots\right)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

=1+0

=1.

Definition 3.2 Let if $\alpha \in (n, n+1]$, and f be an n-differentible at t, where $t > 0$ then the new definition of fractional derivative of f of order α is defined as

$$T_\alpha f(t) = \lim_{\varepsilon\to 0}\frac{f^{(\lceil\alpha\rceil-1)}(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{(\lceil\alpha\rceil-\alpha)} - f^{(\lceil\alpha\rceil-1)}(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Where $\lceil\alpha\rceil$ is the smallest integer greater than or equal to α

Remark 3.1: one can easily prove that,

$$T_\alpha f(t) = t^{(\lceil\alpha\rceil-\alpha)}f^{\lceil\alpha\rceil}(t)$$

Where $\alpha \in (n, n+1]$ and f is n+1 differentiable at $t > 0$

$$T_\alpha f(t) = \lim_{\varepsilon\to 0}\frac{f^{(\lceil\alpha\rceil-1)}(t+e^{-\left|\frac{1}{\varepsilon}\right|}t^{(\lceil\alpha\rceil-\alpha)} - f^{(\lceil\alpha\rceil-1)}(t)}{e^{-\left|\frac{1}{\varepsilon}\right|}}$$

Put h=$e^{-\left|\frac{1}{\varepsilon}\right|}t^{(\lceil\alpha\rceil-\alpha)}$ then $e^{-\left|\frac{1}{\varepsilon}\right|} = h.t^{(\alpha-\lceil\alpha\rceil)}$ as $\varepsilon \to 0$ *then* $h \to 0$

$$T_\alpha f(t) = \lim_{\varepsilon\to 0}\frac{f^{(\lceil\alpha\rceil-1)}(t+h) - f^{(\lceil\alpha\rceil-1)}(t)}{h.t^{(\alpha-\lceil\alpha\rceil)}}$$

$$T_\alpha f(t) = t^{(\lceil\alpha\rceil-\alpha)}\lim_{\varepsilon\to 0}\frac{f^{(\lceil\alpha\rceil-1)}(t+h) - f^{(\lceil\alpha\rceil-1)}(t)}{h}$$

$$T_\alpha f(t) = t^{(\lceil\alpha\rceil-\alpha)}f^{\lceil\alpha\rceil}(t).$$

4. Conclusion

We introduced new generalized definition of conformable fractional derivative.

We conclude that this definition coinside with the classical definition of derivative.

We introduce definition For $\alpha \in (0,1]$ and generalize for any $\in (n, n+1]$.

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